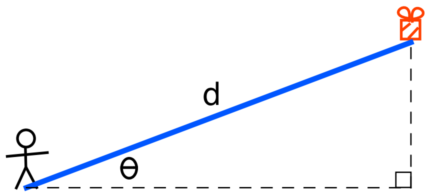
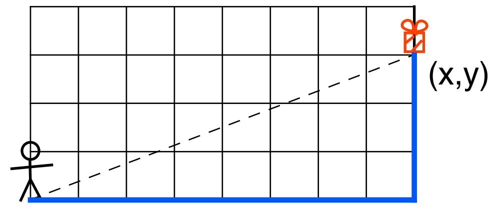
Introduction to Vectors

Imagine you had to tell a friend where to find something (maybe a small gift) in a field. There are two ways you could get them to the gift. You could tell them exactly which way and exactly how far to walk, like in the picture below left. Or you could imagine the field is a big grid, and give them the x and y coordinates of the present, like in the picture below right.





In either case, you would be giving your friend two pieces of information: a distance and a direction (d and θ) or a horizontal distance and a vertical distance (x and y). You just gave your friend a vector!

This year in physics, we will study the mathematical relationships between many things. All of these things are either a *scalar* or a *vector*.

Scalar A quantity that has only a magnitude. Examples are distance, speed, time and mass.

Vector A quantity that has both a magnitude and a direction. Examples are displacement, velocity, acceleration and force.

Magnitude The length of a vector, which tells you how much of the quantity there is. For example, *speed* is the magnitude of *velocity*.

Direction The direction of a vector is usually given as an angle where 0° means pointing to the right, with positive angles going above that and negative angles below.

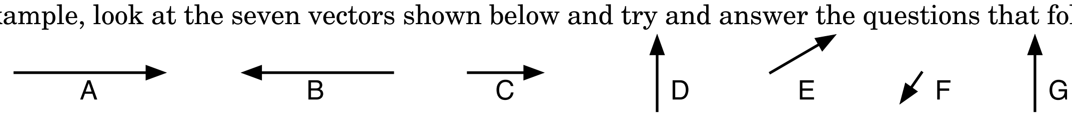
Resultant The sum of two or more vectors.

A good way to think about the difference between these is that a scalar only tells you one piece of information, while a vector gives you two pieces. The scalar *speed* tells you only how fast something is going, but the vector *velocity* tells you how fast and which way something is going.

A. Vector Representation

Vectors are a geometrical object - so it is very easy to just think of a vector as an arrow, pointing in a certain direction and the length tells you the magnitude. Two vectors are equal only if they have the same magnitude and the same direction.

For example, look at the seven vectors shown below and try and answer the questions that follow:



1. Which two vectors have the same magnitude and same direction?

2. What combinations of vectors have the same magnitude, but different directions?

3. What combinations of vectors have the same direction, but different magnitudes?

4. Which vector has the smallest magnitude?

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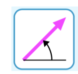
Now to use an online app to explore how to use vectors. Go to https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_en.html and choose "Explore 2D"

Feel free to play with the simulation for a little bit to get the hang of what things do. When you are ready, click on the reset button (the circular arrow in the lower right corner) and continue with the directions below.

B. Understanding Directions of Vectors

Reset the simulation (the circular arrow icon in the lower right) and do the following:

- Clear the screen by unchecking the grid icon in the upper right.
- Show the angles by checking on the angle icon in the upper right (above the grid icon)
- Set the vectors to magnitude and direction by clicking on the pink  vector icon next to the reset icon. (Shown at right.)
- Add a vector by clicking and dragging over one of the pink vectors.

1. Adjust the vector to make the following angles. (The magnitude doesn't matter.) For each angle, sketch what the vector looked like.
   a. 0° b. 45° c. 90° d. –30° e. –90° f. 20° g. 120°

2. What range of values for the direction result in the vector pointing up and to the right?

3. What range of values for the direction result in the vector pointing down and to the right?

C. Understanding Addition of Vectors

Reset the simulation and do the following:

- Clear the screen by unchecking the grid icon in the upper right.
- Drag over two vectors, **a** and **b**, leaving some space in between them.
- Show the sum of the vectors by checking of the "Sum" box in the upper right.

1. There should now be three vectors on the screen. Notice that you cannot directly change the sum, but that if you change vector **a** or **b**, the sum does change. (Hopefully that makes sense – if you change the things you are adding, then the result should also change.)

2. What has to be true for the two vectors to add up to 0? Worded another way, what has to be true for two vectors to cancel each other out? To figure this out, play with the vectors until you can make the sum go to 0.

3. Make vector **a** longer than **b** and give it a direction of 0°. Give vector **b** a direction of 90°. By moving the vectors around, you should be able to make 2 different triangles. Sketch them below – including the labels.

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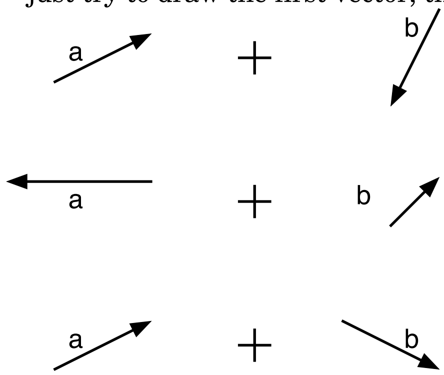
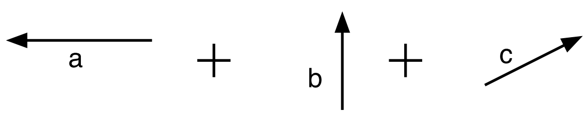
4. Both of the triangles you just found are the two ways you can represent vector addition. Try and add the following pairs of vectors and use the simulation to check. There are no numbers for this – just try to draw the first vector, then the second vector and find the resultant.

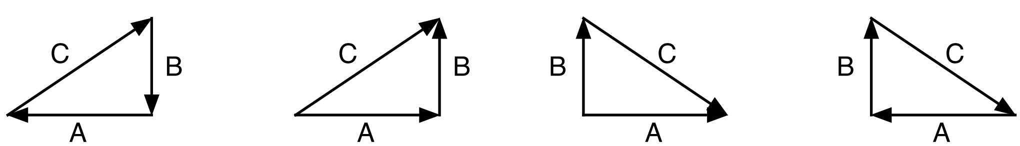


5. Describe how you would take two vectors and add them. (Think about the order that you draw the vectors and where you draw them and how you draw the sum.)

6. While we won't usually add more than two vectors in class, let's see if this is making sense. Drag over vector **c** into the simulation. Add the following three vectors, and then use the simulation to check your answer.



7. The pictures below each show three different vectors, **A**, **B** and **C**. Two of those diagrams show two of the vectors adding up to the third and two of the diagrams show all three vectors adding up to 0. Under each diagram, state what is happening. (Look carefully at how the vectors are drawn.)



D. Understanding Components of Vectors

Reset the simulation and do the following:

- Drag over vector **a**.
- On the right side, under "Components," there are four icons. Click on the upper right one, (shown to the right.)

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1. What happened when you turned on the components? Sketch what you see:

2. Change the vector by giving it a variety of lengths and directions – in fact make sure you spin the vector all the way around. Also look at the top of the screen. You will see a box labeled a_x and another one marked a_y. The simulation is showing you the components of vector a. What do you think is meant by the components of a vector?

3. If a vector is pointed up, which component must be positive?

4. If a vector is pointed to the right, which component must be positive?

5. If a vector is pointed down, which component must be negative?

6. If a vector is pointed to the left, which component must be negative?

x. If a vector is pointed straight up (ø = 90°) which component must be 0?

x. If a vector is pointed just to the right (ø = 0°) which component must be 0?

7. Notice how the components of the vector always make a right triangle, and the actual vector is the hypotenuse. When you are dragging the end of the vector around to change it, you are setting the components of the vector, which is why they are always nice round numbers. The simulation then calculates the magnitude. How do you think it does that? Test it and see on a vector or two.

8. Sketch and label the following vectors and also calculate their magnitudes. Please use one color for the components and a different color for the vector itself. (Use the simulation to check your answers.)

$a_x = 5$	$b_x = 20$	$c_x = 6$	$d_x = 5$
$a_y = 7$	$b_y = 5$	$c_y = 8$	$d_y = -7$
$\lvert\mathbf{a}\rvert$ = ________	$\lvert\mathbf{b}\rvert$ = ________	$\lvert\mathbf{c}\rvert$ = ________	$\lvert\mathbf{d}\rvert$ = ________

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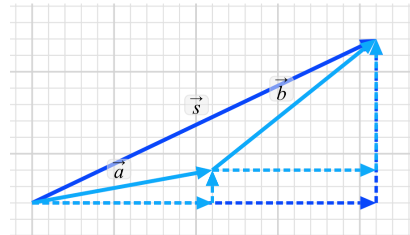
9. It also very easy to add vectors using the components. Drag over vector **b** into the simulation and place it so that it starts at the end of vector **a**. Show the vector sum by checking on the box marked "Sum." Move the blue sum so that it makes the triangle with vectors **a** and **b**. (It should look something like the picture to the right.) How do you add vectors if you know the components? To help you figure this out, change vector **b** (but leave the start of it at the end of **a**)



10. For each part, add the given vectors, and then calculate the magnitude of the vector sum.

a. ($a_x = 15$ & $a_y = 20$) + ($b_x = 5$ & $b_y = 10$) | **a** + **b** | =

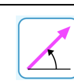
b. ($a_x = 30$ & $a_y = -10$) + ($b_x = 15$ & $b_y = 25$) | **a** + **b** | =

c. ($a_x = 7$ & $a_y = 13$) + ($b_x = 2$ & $b_y = 5$) | **a** + **b** | =

E. Finding Components of Vectors

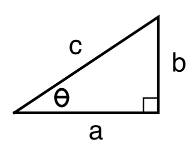
Reset the simulation and do the following:

- Click on the pink vector icon just to the left of the reset icon (shown to the right.)
- Drag over vector **d**.
- On the right side, under "Components," there are four icons. Click on the upper right one, (shown to the right.)



With the pink vectors (**d**, **e** and **f**), the simulation lets you change the magnitude and direction of the vectors, which is why those are nice round numbers at the top of the screen. It then calculates the components using some simple trigonometry.

The two ways of describing a vector (magnitude & direction or x & y components) are related to each other because they make a right triangle. The picture to the right shows a right triangle with sides a, b and c as shown in the diagram. There are three equations from math that you need to remember:



$$c^2 = a^2 + b^2 \qquad \sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c}$$

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1. Currently, the simulation has one vector labeled **d** and it calls the components d_x and d_y. The magnitude of **d** is written as |**d**|, but it is ok to also just write it as "d." Using d, d_x and d_y, rewrite the three math equations:

2. For the following magnitudes and directions, make a sketch that shows the vectors and its components. Then, calculate the components. Use the simulation to check your answers. If you need help using your calculator, call over your teacher.

   a. $d = 20$ & $ø = 30°$ d_x = ___________ d_y = ___________

   b. $d = 11$ & $ø = 55°$ d_x = ___________ d_y = ___________

   c. $d = 15$ & $ø = –25°$ d_x = ___________ d_y = ___________

   d. $d = 4$ & $ø = 40°$ d_x = ___________ d_y = ___________

   e. $d = 7$ & $ø = 90°$ d_x = ___________ d_y = ___________

   f. $d = 18$ & $ø = 0°$ d_x = ___________ d_y = ___________